Weihnachtsworkshop on Geometry and Number Theory

Karlsruhe, December 13 to 15, 2023

Abstracts

Daniele Agostini. *Algebraic Geometry at Sea*

Smooth algebraic curves give rise to solutions to the KP-equation, which models waves in shallow water, via Riemann's theta function. Singular curves produce solutions as well, but the theta function in this case becomes degenerate.
I will give an introductionto this circle of ideas and present some theoretical and computational results on this topic, based on both smooth and singular curves.

Frederik Benirschke. *Bi-algebraic geometry of strata in genus zero*

Strata of differentials are moduli spaces for Riemann Surfaces with meromorphic differentials of fixed zero and pole order. The stratum is an algebraic variety but also has a transcendental period map, obtained by integrating the differential over a homology cycle.
In bi-algebraic geometry, one tries to understand which algebraic subvarieties of strata are described by algebraic equations on the periods. These are called bi-algebraic varieties.
In this talk, I will describe the bi-algebraic subvarieties for strata in genus zero, showing that they are close to being linear in period coordinates.

Felix Clemen. *Combinatorial Geometry: Triangles in the Plane*

A classical problem in combinatorial geometry, posed by Erdős in 1946, asks to determine the maximum number of unit segments in a set of n points in the plane. Since then a great variety of extremal problems in finite planar point sets have been studied. Here, we look at such questions concerning triangles. Among others we answer the following question asked by Erdős and Purdy almost 50 years ago: Given n points in the plane, how many triangles can be approximate congruent to equilateral triangles?
For our proofs we use hypergraph Turán theory.
This is joint work with Balogh and Dumitrescu.

Nguyen-Thi Dang. *Equidistribution of tori for the geodesic flow in higher rank*

Bowen and Margulis independently proved in the 70s that closed geodesics on compact hyperbolic surfaces equidistribute towards the measure of maximal entropy. From a homogeneous dynamics point of view, this measure is the quotient of the Haar measure on $\mathrm{PSL}(2,\mathbb{R})$ modulo some discrete cocompact sugroup.
In a joint work with Jialun Li, we investigate the higher rank setting of this problem by taking a higher rank Lie group (like $\mathrm{SL}(d,\mathbb{R})$ for $d \geq 3$) and by studying the dynamical properties Weyl chamber flows: geodesic flows in higher rank. We obtain an equidistribution formula of periodic tori (instead of closed orbits of the geodesic flow).

Samantha Fairchild. *Mean value theorems for the S-arithmetic primitive Siegel transforms*

The Siegel mean value formula is a classic result in the geometry of numbers, which says the expected number of lattice points in the ball is the size of that ball. When reducing to primitive lattice points (points which are visible from the origin), the primitive Siegel mean value formula says the expected number of lattice points in the ball is the size of that ball scaled by the density of primitive vectors. We will discuss the theorems of Siegel, and the variance estimates obtained by Schmidt and Rogers, and see what happens when we consider these same questions over lattices which also include a component over the p-adic numbers.

Magali Jay. *Tiling billiards and interval exchange transformations with flips*

Let us consider a tiling of the Euclidean plane by polygons. We play the billiard on this tiling in the following way. A trajectory goes in straight line in each tile. When it reaches a boundary between two tiles, it crosses the boundary and is refracted in the new tile. We get a zigzaging trajectory in the plane. Our goal is to understand what behaviour it can have: Is it periodic or not? bounded or not? If not, how does it go to infinity?
I will first present this dynamical system, and explain how to study it with interval exchange transformations with flips, which are piecewise isometries of the circle.
This will allow me to state a result of my PhD thesis about deviations from asymptotic direction of some tiling billiards trajectories and to explain its proof.

Pascal Kattler. *Kontsevich-Zorich monodromy of origamis of genus 2*

Origamis are a special kind of translation surface. We will introduce the Kontsevich-Zorich monodromy of origamis, which is a subgroup of $\mathrm{SL}_2(\mathbb{Z})$. Then we will show that the index of it in $\mathrm{SL}_2(\mathbb{Z})$ is bounded for all origamis in the stratum $\mathcal{H}(2)$.

Carlos Matheus. *Exotic bi-algebraic subvarieties of moduli spaces of Abelian differentials*

In their recent work, Klingler and Lerer took the bi-algebraic point of view on the moduli spaces of Abelian differentials in order to study the transcendental features of the periods of these objects. In particular, Klingler and Lerer showed that all bi-algebraic curves in moduli spaces of Abelian differentials of genus two are linear and, more generally, this is still the case in any genus provided their condition (*) is fulfilled.
In this talk, we discuss a joint work with Deroin describing non-linear bi-algebraic curves and surfaces in moduli spaces of Abelian differentials of high genus.

Preda Mihailescu. *Binomial series, Catalan's equation ... and further Christmas tales*

After a historical introduction on the different phases of progress on the proof of Catalan's conjecture, I shall present a new, shorter proof, which also uses binomial series, but instead avoids Thaine's theorem.
As far as time allows, the ... further Christmas tales shall speak about further developments of the ideas in local diophantine approximation.

Martin Möller. *Spectral decomposition and Siegel-Veech transforms*

The spectral decomposition of square integrable functions on the modular surface is well-

studied. Much less is known about the spectral decomposition for moduli spaces of flat surfaces. For the case of the torus with marked points we present a complete picture, featuring Siegel-Veech transforms as generalizations of Eisenstein series.

Hugo Parlier. *Crossing the line: from graphs to curves*

The crossing lemma for simple graphs gives a lower bound on the necessary number of crossings of any planar drawing of a graph in terms of its number of edges and vertices. Viewed through the lens of topology, this leads to other questions about arcs and curves on surfaces. Here is one: how many crossings do a collection of m homotopically distinct curves on a surface of genus g induce?
The talk will be about joint work with Alfredo Hubard where we explore some of these, using tools from the hyperbolic geometry of surfaces in the process.

Maurice Reichert. *Counting connected components in the phase space of linear trajectories on the octagon*

John Smillie and Corinna Ulcigrai have constructed transition diagrams for admissible words on the symbolic coding of linear trajectories on the regular octagon. Our initial objective was to count these admissible words up to a given length. Unexpectedly, two different computational approaches yield distinct results, revealing a fascinating discrepancy. This difference can be attributed to the varying number of connected components within the phase space of the linear flow on the octagon and depends on the specific admissible word employed to describe the trajectory. I will give an explicit formula for quantifying the connected components of all admissible words.
This is based on joint work with Jayadev S. Athreya, Nicolas Bédaride, and Julien Cassaigne.

Christopher Voll. *Groups, graphs, hypergraphs – average sizes of kernels of generic matrices and conjugacy classes of finite p-groups*

A fundamental invariant of a finite group is the number of its conjugacy classes or, equivalently, its irreducible complex characters. The determination of this "class number" is, in general, a difficult problem. The variation of class numbers in natural families is the object of central conjectures in the theory of finite p-groups, such as G. Higman's famous conjecture on the polynomiality of class numbers of full unitriangular matrix groups over finite fields.
I report on joint work with Tobias Rossmann. We develop a theory of average sizes of kernels of generic matrices with support constraints encoded by graphs and hypergraphs. As an application of this theory, we prove strong uniformity theorems about the variation of the class numbers of finite "graphical groups", certain finite p-groups associated with graphs in a canonical way. These uniformity (i.e. polynomiality) theorems are in striking contrast to algebro-geometric results of Belkale and Brosnan about the rank distribution of symmetric matrices with support conditions over finite fields: the exact rank distribution is – in a precise, technical sense – arbitrarily wild.
Methodologically, our project combines ideas from graph and group theory, but also from toric geometry and p-adic integration theory, and has strong algorithmic aspects. (https://arxiv.org/abs/1908.09589)

Yiu Man Wong. *An algorithm to compute the fundamental classes of spin components of strata of differentials*

The connected components of strata of differentials of even type can be bipartitioned according to their spin parities. One can consider the projectivized strata as subvarieties on $M_{g,n}$ and investigate the closure of it in $\bar{M}_{g,n}$. The fundamental classes of closures of the spin components on $\bar{M}_{g,n}$ allow us to reduce the computations of intersection theory on spin components of projectivized strata to intersection on $\bar{M}_{g,n}$.

In this talk, we will introduce an algorithm to compute these fundamental classes. Our main strategy is to reconstruct these cycles by their restrictions to the boundary of $\bar{M}_{g,n}$ via clutching maps.