

Weihnachtsworkshop on Geometry and Number Theory

Karlsruhe, December 15 to 17, 2025

Abstracts

Konstantin Bogdanov. *Infinite-dimensional Thurston theory and holomorphic dynamics*

Thurston's Topological Characterization of Rational Functions is a criterion which tells us when exactly a topological model of a rational function with finite orbits of critical values is realized as a genuine holomorphic rational function on the Riemann sphere. The theorem is one of the most fundamental results in holomorphic dynamics. A key to the proof is to study the associated pull-back map σ on the Teichmüller space of the Riemann sphere with finitely many punctures (at the critical orbits).

However, if we allow infinite critical orbits, the map σ , if at all defined, acts on an infinite-dimensional Teichmüller space and the classical argument is no longer applicable, which leads us to an independent theory. During the talk I will overview the existing results which hold also for large families of transcendental entire functions and explain the difference between the finite- and infinite-dimensional cases.

Thomas Dubbe. *Applications of k -dimensional sieves*

In this talk, I will give a brief introduction to sieve theory, paying particular attention to the large sieve. Armed with the necessary tools, we will then explore two applications. The first stems from finite automata and concerns the q -automaticity of the set of prime numbers. The second is about prime numbers p , such that the shifted prime $p - 1$ has at least one large prime factor $> p^\theta$, where $0 < \theta < 1$.

Felix Cherubini. *Synthetic Algebraic Geometry*

Looking at the functor of points approach to algebraic geometry, one might be tempted to drop some indices of, say, a natural transformation between sheaves and pretend that we just have a function between sets, which we can apply to elements. Synthetic Algebraic Geometry allows for exactly this and many more simplifications arising from the use of what is called an internal language of the category of Zariski-sheaves. Since we use Homotopy Type Theory as a basis for such an internal language, we have a concise approach to algebraic stacks and cohomology of schemes.

In the talk, I will show what it is like to prove basic facts about projective space in this language and make some remarks on what has been proven so far in this style. This is joint work with Thierry Coquand, Matthias Ritter, Hugo Moeneclaey and David Wärn.

Sebastian Engelhardt. *The dessins d'enfants stemming from boundary points of Teichmüller curves of origamis in $\mathcal{H}(2)$*

Origamis are translation surfaces that consist of finitely many Euclidean squares glued together along their edges. By deforming its translation structure, an origami defines a special kind of Teichmüller curve. These are complex algebraic curves in the moduli space \mathcal{M}_g of closed complex regular curves of genus g . In this talk, we study the boundary points of Teichmüller curves stemming from origamis in the stratum $\mathcal{H}(2)$ on the Deligne–Mumford

compactification of \mathcal{M}_2 . Stable reduction is a general method for finding boundary points of algebraic curves in \mathcal{M}_g , represented by stable Riemann surfaces. For origami curves, the method can be described via contraction of the core curves of horizontal cylinders of the origami. Each irreducible component of the resulting stable surface naturally gives rise to a dessin d'enfant—a bipartite graph embedded into a Riemann surface. We give a graph-theoretical characterization of all occurring dessins for origamis in $\mathcal{H}(2)$.

Erick Gordillo. *Quantitative weak mixing property for linear involutions*

Linear involutions are one-dimensional dynamical systems that generalize interval exchange transformations, naturally arising from non-orientable foliations on orientable surfaces. Avila and Forni proved that interval exchange transformations are generically weakly mixing, and later, Avila, Forni, and Safaee established a quantitative version of this result, demonstrating polynomial decay of correlations for a suitable class of observables.

In this talk, I will present an extension of these results to the non-orientable setting, showing that weak mixing remains generic for linear involutions, with the same polynomial decay of correlations.

Elise Goujard. *Volumes of odd strata of quadratic differentials*

I will present a formula giving the Masur-Veech volumes of “completed” odd strata of quadratic differentials as a sum over stable graphs. This formula generalizes Delecroix-G-Zograf-Zorich formula in the case of principal strata. The coefficients of the formula are in this case intersection numbers of ψ -classes with the Witten-Kontsevich combinatorial classes; they naturally appear in the count of integer metrics on ribbon graphs with prescribed odd valencies. The study of the possible degenerations of these ribbon graphs allows to express the difference between the volume of the “completed” stratum and the volume of the stratum as a linear combination of volumes of boundary strata, with explicit rational coefficients. Several conjectures on the large genus asymptotics of volumes or distribution of cylinders follow from this formula. (joint work with E. Duryev and I. Yakovlev).

Fabian Januszewski. *Cohomology of arithmetic groups and integral structures on automorphic representations*

In this talk I will discuss various aspects of rational and integral structures on spaces of automorphic forms of reductive groups originating in the cohomology of arithmetic groups. A key concept behind the construction of rational and integral structures is the notion of locally algebraic representation which gives rise to locally algebraic (\mathfrak{g}, K) -modules as well as to associated sheaves on associated arithmetic orbifolds. These notions overcome certain conceptual obstacles in the standard treatment of arithmetic groups, their cohomology, and associated automorphic representations and align with Langlands’ philosophy of parametrizing L -functions by L -group representations.

Vítězslav Kala. *Universal quadratic lattices and Northcott property of infinite number fields*

Universal quadratic forms and lattices generalize the sum of four squares about which it is well known that it represents all positive rational integers. In the talk, I’ll start by discussing some results on universal quadratic lattices over totally real number fields. Then I’ll move on to the - markedly different! - situation over infinite degree extensions K of \mathbb{Q} .

In particular, I’ll show that if K doesn’t have many small elements (i.e., “ K has the Northcott property”), then it admits no universal lattice. Further, almost all totally real fields do not have universal lattices or the Northcott property (in a suitable topological sense).

The talk is based on recent joint works with Nicolas Daans, Siu Hang Man, Pavlo Yatsyna, and Martin Widmer.

Claudio Llosa. *The Boone–Higman Conjecture for groups acting on locally finite trees*

The Boone–Higman Conjecture asserts that a finitely generated group has solvable word problem if and only if it embeds in a finitely presented simple group. While it has recently been confirmed for many interesting classes of groups, including hyperbolic groups and virtually special groups, it remains wide open in general. In this talk we explain a method for proving the Boone–Higman Conjecture for groups acting on locally finite trees, which may offer a route for proving it for many new classes of groups. We illustrate our method by showing the Boone–Higman Conjecture for all free-by-cyclic groups and all Baumslag–Solitar groups, solving it in two cases that have been raised explicitly by Belk, Bleak, Matucci and Zaremsky. This is joint work with Kai-Uwe Bux and Xiaolei Wu.

Bianca Marchionna. *Topological right-angled Artin groups*

Right-angled Artin groups (RAAGs) form a family of finitely generated groups that play an important role in geometric and computational group theory. Their study is often related to the one of their universal Salvetti complex, a CAT(0)-cube complex on which they act geometrically.

In this talk, we introduce a generalisation of RAAGs to topological groups. Remarkably, those groups have a cellular action on a thicker version of the universal Salvetti complex of a RAAG with controlled cell stabilisers. Due to the high connectivity of this complex, we can deduce homological/homotopical finiteness properties for topological RAAGs.

Work in progress with I. Castellano, B. Nucinkis, and Y. Santos Rego.

Nicole Müller. *Teichmüller curves and modular forms*

Teichmüller curves are special algebraic curves $C \subset \mathcal{M}_g$ inside the moduli space of curves; e.g. the image of the projection from the moduli space of flat surfaces of the $\mathrm{SL}_2(\mathbb{R})$ -orbit of a Veech surface. We will focus on a special class of Teichmüller curves arising from Hecke triangle groups and embed them into the locus of real multiplication inside \mathcal{A}_g , the moduli space of abelian varieties. Furthermore, we will describe the “Heptagon” Teichmüller curve, which is generated by the double-heptagon surface, as the vanishing locus of Hilbert modular forms. This is an extension of the work of Möller-Zagier for Teichmüller curves generated by a genus 2 Veech surface to genus 3.

Sophie Schmidhuber. *Dynamical decomposition of foliations on dilation surfaces and affine interval exchange transformations*

Dilation surfaces are surfaces modeled after the complex plane whose structure group is generated by the group of translations and dilations. Given a dilation surface, for any direction in S^1 there exists a corresponding directional foliation on the surface. In this talk, we will study the four possible types of dynamical behaviour that such a foliation may have (i.e completely periodic, Morse-Smale, minimal or Cantor-like) and deduce a dynamical decomposition theorem for the directional foliation on dilation surfaces using results of C.J. Gardiner and G. Levitt from the 1980s.

In a second step, we study the first return map of the directional foliation on a dilation surface, which is a so-called affine interval exchange transformation (AIET). We introduce a powerful tool called Rauzy-Veech induction in order to develop a renormalization scheme which allows to find a decomposition of any given AIET into a finite union of intervals which

exhibit only one of the four types of dynamical behaviour. This provides an alternative, purely combinatorial approach to the decomposition results of Levitt and Gardiner and is joint work with Corinna Ulcigrai and Charles Fougeron.

Jeroen Sijsling. *Modularity in genus 2*

Given an elliptic curve E over the rational numbers, the modularity theorem shows that its L -function $L(E, s)$ can equally well be realized as an L -function $L(f, s)$ associated to a modular form of weight 2 for $\mathrm{GL}_{2, \mathbb{Q}}$ with rational coefficients. This association gives rise to a bijection between the set of isogeny classes of elliptic curves E over \mathbb{Q} of a given conductor N and the set of normalized newforms $f \in S_2(\Gamma_0(N), \mathbb{Q})$.

This talk discusses how these results generalize (at least conjecturally) to curves of genus 2. Given such a genus 2 curve X over a number field F , we start by exploring the information contained in the L -function $L(J, s)$ of its Jacobian $J = \mathrm{Jac}(X)$. Things get considerably stranger, and in particular the above bijection may break down because of the presence of false elliptic curves, so that we have to state carefully where to look for a modular form f for $\mathrm{GL}_{2, F}$ associated to J , if indeed any such form exists at all.

If the endomorphism ring of J is a quadratic number field M (so that J is of so-called $\mathrm{GL}_{2, F}$ -type), then a conjecture by Ribet claims the existence of a modular form f for $\mathrm{GL}_{2, F}$ with coefficient field M such that $L(J, s) = L(f, s)$. In general, it turns out that there always exists a quadratic extension F' of the base field F of X such that J is of $\mathrm{GL}_{2, F'}$ -type over F' . A consideration involving restrictions of scalars then shows that we have $L(J, s) = L(f, s)$, where f is a modular form for $\mathrm{GL}_{2, F'}$ associated to the base extension of J to F' ; moreover, the property that said base extension actually comes from a curve over F (namely X) is reflected in a peculiar property of the modular form f that we call Galois alignment.

We illustrate these constructions by means of concrete examples. This is joint work with Andrew Booker, Andrew Sutherland, Dan Yasaki, and John Voight.

Yuriy Tumarkin. *Invariant sets in the wind-tree model*

The wind-tree model is a billiard model of gas diffusion introduced by Ehrenfest and Ehrenfest in the early 20th century. A small particle (the "wind") bounces between periodically placed rectangles (the "trees") on the plane. A lot of progress has been made in the past 15 years by studying the wind-tree model from the point of view of translation surfaces. In particular, Fraćzek and Ulcigrai proved in 2014 that in typical directions the billiard flow on the wind-tree model has no dense trajectories. This opens the natural question of trying to understand what the closures of the trajectories look like.

The new result that I will present in this talk is that in the self-similar case, all closures of trajectories have non-maximal Hausdorff dimension.